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SAFİZMLERLE BİR ÖĞRENME ORTAMI TASARLAMA: ÖĞRETMENLERİN ÖĞRENCİLERİN İLGİSİNİ ÇEKMESİ VE ONLARIN MATEMATİK DERSLERİNE KATILIMINI GELİŞTİRMESİ İÇİN ÖNERİLER

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Özet

Matematiğin keşfi genellikle olağanüstü zorluklarla, karmaşık bulmacalarla, karmaşık zihinsel oyunlarla, paradokslarla ve düşündürücü sofizmlerle karşılaşmayı içerir. Büyüleyici örneklere dalmak, öğrencileri meşgul etme, aydınlatma ve ilham verme potansiyeline sahiptir ve keşfetme isteğini teşvik eder. Bu araştırma makalesi, bazı ilgi çekici matematiksel sofizmleri ve bunların matematik eğitimi alanındaki etkilerini açıklamayı amaçlamaktadır. Birçok faktör, matematiği hümanistik bir bakış açısıyla öğretmek için gelişen bir eğitim ortamının kurulmasını etkiler. Bu makalede benimsenen metodoloji, bir çoklu durum çalışması ile belirli örnekleri göstermeyi, bu özellikleri vurgulamayı ve mantıksal tutarsızlıklara yol açabilecek varsayımlarına sıkı sıkıya bağlı kalmadan matematiksel kavramları uygulamayı içerir. Öğrenciler, bir sayıyı sıfıra bölme veya negatif olmayan bir karekökü dışarı çıkarma gibi konularda sıklıkla yardıma ihtiyaç duyarlar. Sofizmlere yol açan hatalı düşünme teknikleri ve aşamaları, karekökler, trigonometri, denklemler, türev alma, logaritmalar, geometri, binom açılımı ve integral gibi çeşitli matematiksel ilkelerle karmaşık bir şekilde bağlantılıdır. Çalışma sonucunda, neşeli hislerle sağlıklı bir öğrenme ortamı geliştirmenin gereklilikleri ifade edilmiştir. Sonuç olarak, bu duygular öğrencilerin düşüncelerini harekete geçirecek, bilişsel ilgi alanlarıyla bağlantı kuracak ve gelecekteki girişimlerinde onlara yardımcı olacaktır. Bu araştırma, bu matematiksel kavramların her biriyle ilişkili sofizmleri sunmaktadır. Matematiğe ilgi duyanların bu makalede sunulan kavramları gelecekteki matematiksel araştırmalar için bir katalizör olarak algılamaları umulmaktadır.

Anahtar Kelimeler: Eleştirel düşünme, katılım, kavram yanılgıları, öğrenme ortamı, sofizmler, STEM eğitimi

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DESIGNING A LEARNING ENVIRONMENT WITH SOME SOPHISMS: RECOMMENDATIONS FOR TEACHERS TO PIQUE STUDENTS' INTEREST AND IMPROVE THEIR ENGAGEMENT IN MATHS SESSIONS

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Abstract

The exploration of mathematics often involves encountering extraordinary challenges, intricate puzzles, sophisticated mental games, paradoxes, and thought-provoking sophisms. Delving into captivating examples has the potential to engage, enlighten, and inspire students, fostering a drive for discovery. This research paper aims to elucidate some intriguing mathematical sophisms and their implications within the realm of mathematics education. Several factors impact the establishment of an evolving educational setting for teaching mathematics from a humanistic viewpoint. The methodology adopted in this paper is a multi-case study that involves showcasing specific examples, highlighting these characteristics, and implementing mathematical concepts without strictly adhering to their assumptions, which can lead to logical inconsistencies. Students frequently need help with topics such as dividing an equation by zero or extracting a nonnegative square root. The tactics and stages of erroneous thinking that give rise to sophisms are intricately linked to various mathematical principles, including square roots, trigonometry, equations, differentiation, logarithms, geometry, binomial expansion, and integration. The requirements for developing a healthy learning environment with cheerful sensations are undoubtedly expressed. As a result, these emotions will stimulate students' thinking, link with their cognitive interests, and aid them in their future undertakings. This exploration delves into sophisms associated with each of these mathematical notions. It is our aspiration that those with an interest in mathematics will perceive the concepts presented in this article as a catalyst for future mathematical research.

Keywords: Critical thinking, engagement, learning-environment, misconception, sophisms, STEM education

1. INTRODUCTION

Mathematical operations and expressions are employed in various contexts, ranging from fairs to scientific research. Nevertheless, its users sometimes need to pay more attention to its uses since they are outside the scope of education (Castro & Pereira, 2020). However, utilizing particular mathematical approaches without proper attention to the essential hypotheses for such applications can frequently result in very contentious findings. It is possible to derive inconsistencies or terrifying consequences by utilizing reasons that are not legitimate in a resolution or demonstration, such as those that harm axioms, logical reasoning, or definitions. These arguments are often purposefully extended to deduce incorrect conclusions from accurate premises. This form of thinking is known as a sophism in this circumstance.

Understanding and retaining new mathematical information is crucial to developing pupils' abilities to use this knowledge. Developing planned mathematical competencies measures the efficiency of mathematics instruction in a specific class. Furthermore, the desire to perform a

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certain action and the emotional state of the subject while performing the activity influence instructional efficiency (Ganchev, 1999).

So far, we have looked at numerous definitions of mathematics, such as “Mathematics is referred to as the science of logical reasoning” and “Mathematics is a means of instilling a habit of reasoning in the mind.” Without a doubt, mathematics plays an important function in mind training, providing actual disciplinary value and developing a feeling of order in cognitive processes. When the teaching-learning process is done well, it promotes logical reasoning and improves cognitive abilities.

Aside from these definitions, the history of mathematics is littered with startling and intriguing sophisms that led to the creation of new definitions within the topic. As a result, distinguishing between captivating and instructional sophisms and paradoxes is critical. The primary goal of this paper is to investigate sophism.

The name “sophism” (σόφισμα) comes from the Greek word “Sophos,” which means “wisdom.” In modern parlance, it refers to purposely flawed reasoning that seems nominally right but contains a minor error or fault. A sophism is essentially a fake proof of an inaccurate proposition, with each such “proof” having some type of logical mistake. Plato denounced Sophists who used purposefully false arguments for selfish benefit in his pursuit of truth. In “Calculus: A Liberal Art” (Priestley, 1998) provides an interesting summary of this history.

In addition, The word “paradox” (παράδοξο) comes from the Greek word “paradoxon,” which means “surprise.” It has a variety of applications, some of which allow for conflicts. However, the term “paradox” shall be used exclusively in this paper to refer to a surprise, unexpected, paradoxical assertion that looks invalid but is, in fact, accurate. The remark “All I know is that I know nothing,” ascribed to Socrates in Plato's Republic, is an example of a classical logic paradox.

Exploration of non-routine issues, puzzles, paradoxes, and sophisms typically generates excitement and intrigue in the field of mathematics. Exciting examples can inspire, illuminate, and encourage students, kindling their desire to learn. Engaging students in thought-provoking tasks and encouraging them to consider paradoxes may also naturally engage them, offering a unique opportunity to get a more thorough grasp of the history and development of mathematics. “Justification of otherwise inexplicable notions because they yield useful results has occurred frequently in the evolution of mathematics” (Kleiner & Movshovitz-Hadar, 1994).

The interdisciplinary approach to solving real-world problems and establishing connections with different disciplines, known as STEM, is highly favored in 21st-century education. Among the subtopics of this model, Mathematics holds a particular significance. Our research topic can be addressed with this approach as well.

The need to make STEM (science, technology, engineering, and mathematics) education more appealing at all levels of schooling is one of the key educational controversies (Winberg et al., 2019). However, research has revealed that while many students are capable of dealing with everyday challenges, some are unable to deal with real-world issues because they lack critical and creative thinking skills. Students who are forced to answer textbook problems using the methods taught in class must master the problem-solving strategies needed to deal with real-world circumstances (Michalewicz, 2008). One method for overcoming this issue is active learning.

Active learning is a teaching technique that “involves students in the process of learning through activities and discussion in class, rather than passively listening to an expert.” It emphasizes higher-order thinking and typically includes group projects” (Freeman et al., 2014). Students can participate in additional mathematical studies, collaborate to solve problems, and increase their STEM course skills (Lugosi & Uribe, 2022). They offered six active learning strategies, including group work with discussion and feedback, raising students' passion for curricular themes, and incorporating students with mathematical discoveries, experiments, and projects (2022). Problem-based learning is one of the learning approaches that may help to students' active learning (Lambros, 2002).

When there is a lack of appropriate intellectual challenges and insufficient active participation or emotional commitment from students, the efficacy of the teaching and learning process typically suffers. Knowledge and learning experiences connected with serious thought, important discovery, or inspired creativity tend to leave a lasting imprint. This paper provides solved examples that are intended to stimulate students to ponder conceptual challenges in calculus and obtain a more in-depth grasp of the subject's complexities.

Throughout their schooling, many students come across sophisms. Identifying and evaluating the error in a sophism frequently leads to a deeper knowledge than a merely formulaic approach to problem-solving. The paper's goal is to improve the teaching and learning experience in math courses. It looks into essential subjects generally taught in a single-variable math course, as well as certain fundamental concepts. This article focuses on a specific type of example: Sophism. We'll also briefly discuss paradoxes. Why should we attempt to investigate these odd or difficult math problems? Take a look at the following interesting thoughts from “How Mathematicians Think: Using Ambiguity, Contradiction, and Paradox to Create Mathematics” (Byers, 2007): “While logic rejects ambiguity, paradox, and especially contradiction, creative mathematicians embrace such problematic situations because they elicit the question, 'What is going on here?'“ As a result, the problem suggests a scenario that requires more examination. The issue is a possible source of new mathematical ideas.”

2. LITERATURE REVIEW

Mathematics is present in people's daily lives, whether it is formalized in a school or academic context or informalized in circumstances where formalities are not always observed. When mathematical ideas are applied carelessly, they may yield results that do not satisfy or represent reality. It was demonstrated how to generate mathematical absurdities using basic assertions that the majority of the students were familiar with. Some of the proposed solutions were implemented, typically by students who failed to pay attention to the definitions and assumptions necessary for their use (da Silva & de Albuquerque Soares, 2023). A theorem that confirms the obvious has also been developed. As a result, when using faulty mathematical reasoning in a demonstration or application, such as those that break axioms, logical arguments, definitions, or assumptions, it is possible to deduce inconsistencies or alarming implications.

Attitudes and perceptions of mathematics are two aspects that influence students' learning and achievement in mathematics (Nedaei et al., 2019; Sarouphim & Chartouny, 2017). Several factors influence students' attitudes toward and perceptions of mathematics, including academic and social settings at educational institutions, course material, instructional approaches, teachers, and students' experiences with mathematics (Byers et al., 2018; Ellis et al., 2014). Students are more interested in mathematics learning when they understand the value of mathematics in their daily lives (Attard, 2012). Furthermore, students' views toward mathematics have a beneficial influence on classroom involvement. Teachers have a significant impact on students' attitudes toward and views of mathematics. Teachers have a significant impact on students' attitudes toward and views of mathematics. The techniques used to teach mathematics, how they engage directly with students, and how they include them in specific activities in their fields of study may all affect students' attitudes and impressions of mathematics (Hamzeh, 2009; Klingler, 2012).

According to Ganchev's research, (1999) mathematics education should follow five key principles. The following is the explanation of the first criterion: “Once an individual's personality has taken shape through social practice, presenting itself as a unified entity where cognitive activity seamlessly integrates with experience, the cultivation of positive emotions plays a crucial role in the learning process, including the learning of mathematics.” Our analysis focuses primarily on the educational part of this design, outlining the precise requirements for constructing an environment that takes into account the age-related characteristics of students' personalities. This involves their understanding, conceptualization, memory, and application of specific mathematical information, all while stressing pleasant experiences and emotions.

The article (Karakasheva, 2016) discusses the possibility of building a pleasant, emotional, educational environment in mathematics instruction at the early stages of school. Karakasheva (2017) supports the premise that providing a learning environment that elicits happy feelings while teaching mathematics motivates pupils to perform better. The discussion focuses on two methods for producing pleasant emotional learning situations: mathematical tricks and sophisms.

According to research (Rezvanifard et al., 2023), more than half of lecturers and students believe that sophism and paradox exercises are interesting and engaging activities that increase students' grasp of mathematics, problem-solving abilities, and critical thinking skills. The findings are riddled with ambiguity and conflict. Tasks can be used in conjunction with regular classroom problems to encourage students to participate more actively in class discussions and to drive them to learn arithmetic.

PBL (Problem-based learning) is a “pedagogical approach that enables students to learn while actively engaging with meaningful problems” (Yew & Goh, 2016). PBL has several characteristics, including being student-centered, occurring in small groups, teachers acting as facilitators rather than information dispensers, and challenges focusing on stimulating learning and strengthening students' problem-solving abilities (Barrows, 1996; Hmelo-Silver, 2004). The purpose of implementing PBL is to improve students' problem-solving skills as well as their enthusiasm to learn. It also promotes self-directed learning, critical thinking, leadership, successful collaboration, and long-term memory (Arslan, 2010; Klymchuk, 2013).

The goal of Klymchuk and Sangwin's (2021) study is to look at the viewpoints of school mathematics instructors on using provocative mathematics questions in teaching and assessment as a potential educational innovation. A provocative mathematics question is one that is designed to perplex the solver. It frequently requires an impossible task.

Radmehr & Vos's study (2020) casts a unique light on us. In the twentieth century, assessment in mathematics classrooms mainly focused on knowledge with little cognitive demands, notably testing students on memorized calculating skills. Higher-order thinking (HOT) is becoming increasingly essential in the twenty-first century, which should be reflected in math grading. This will need substantial modifications in assessment and instruction, as teachers must carefully evaluate how to prepare children for assessment forms that activate HOT successfully.

The term "sustainability" is frequently connected with environmental and economic difficulties but also relates to education. We must not overlook the effort that will serve as a valuable resource in ensuring the long-term viability of this research. (Joutsenlahti & Perkkilä, 2019) focuses on long-term progress in mathematics instruction from the standpoint of teacher education. The goal was to improve prospective teachers' subject and pedagogical expertise in school mathematics.

3. METHODOLOGY

The methodology of this paper a multi-case study that consists of presenting certain instances, demonstrating these features, and applying these mathematical ideas without paying heed to their hypotheses, resulting in mathematical absurdities.

Humanizing the learning process is a crucial part of modernizing education. Relationships, values, symbols, and objects, among other things, are utilized to build such a process. Various criteria influence the creation of a developing educational environment in teaching mathematics with a humanistic perspective, including:

a) The creation of modern forms of individual and collaborative work by teachers and students that are tailored to student's needs, interests, opportunities for self-expression, and participation in intellectual activities varying in difficulty and content;
b) The preservation and development of each student's creativity;
c) The establishment of clear rules for communication, self-organization, and self-government, as well as the adoption of behavioral standards that ensure inclusion in the classroom.

4. APPLICATIONS

This section contains various applications connected to the sophism that we have tried to talk about so far. While further questions are welcome, we feel that the subject may be adequately grasped with the present diversity of examples spanning many mathematical concepts.

Example 1. Prove that $a = b$, $\forall a, b \in \mathbb{R}$.

Knowing that $a^2 - 2ab + b^2 = b^2 - 2ab + a^2$ is true $\forall a, b \in \mathbb{R}$. So $(a-b)^2 = (b-a)^2$, and take square root of both sides to get $a - b = b - a \Rightarrow 2a = 2b$ dividing both sides by 2 gives us

$a=b$. Anymore instead of a and b substitute whatever you want and get different equalities like $1=2$, $11=19$,…etc.

Example 2. Demonstrate that $2^2=4^2$.

Using the basic trigonometric identity,

$$\begin{aligned}
\cos^2 x + \sin^2 x &= 1 \\
(\cos^2 x)^{\frac{3}{2}} &= (1-\sin^2 x)^{\frac{3}{2}} \\
\cos^3 x &= (1-\sin^2 x)^{\frac{3}{2}} \\
\cos^3 x + 3 &= (1-\sin^2 x)^{\frac{3}{2}} + 3 \\
(\cos^3 x + 3)^2 &= [(1-\sin^2 x)^{\frac{3}{2}} + 3]^2
\end{aligned}$$

Let us examine the ensuing equality for $x=\frac{\pi}{2}$ yields, $3^2=3^2$.

However, when we verify for $x=\pi$ where $\cos\pi=-1$and $\sin\pi=0$, we obtain $2^2=4^2$ (Мадера, 2003).

Example 3. Show that $8=6$.

If we solve the system $\begin{cases} x+2y=6 \\ y=4-\frac{x}{2} \end{cases}$ by using substitution method,

$x+2\left(4-\frac{x}{2}\right)=6 \Rightarrow x+2.4-2.\frac{x}{2}=6$, and after necessary algebraic operation, we get $8=6$.

Example 4. Demonstrate that $\cos^2 x=1,\ \forall x\in\mathbb{R}$.

Let's try to find the given function's second derivative.

$y=\frac{1}{\tan x}$ and $y'=-\frac{1}{\sin^2 x}$ then $y''=\frac{2\sin x\cos x}{\sin^4 x}=\frac{2\cos x}{\sin^3 x}=2\times\frac{1}{\tan x}\times\frac{1}{\sin^2 x}$.

Using the first and second differentiations, we obtain the following relationship:

$y''=-2yy'$ or $y''=-(y^2)'$.

If we integrate both sides of the equation $y'' = -(y^2)'$ leads to $y' = -y^2$ then substitute the values respectively for $y' = -\frac{1}{\sin^2 x}$ and $y^2 = \frac{1}{\tan^2 x}$, so $-\frac{1}{\sin^2 x} = -\frac{1}{\tan^2 x} \Rightarrow \frac{1}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x}$. From here we conclude that $\cos^2 x = 1$ for all real values of x (Мадера, 2003).

Example 5. Show that $\frac{1}{4} > \frac{1}{2}$.

Naturally, $\frac{1}{2} = \frac{1}{2}$ and taking the common logarithm of both sides $\log\frac{1}{2} = \log\frac{1}{2}$

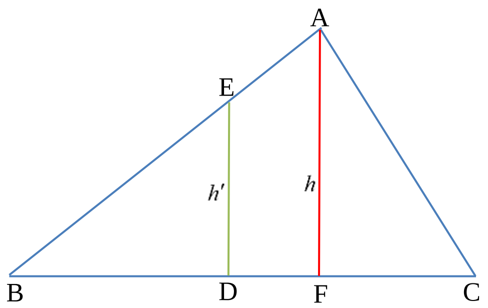
Remember! if $a = a \Rightarrow 2a > a$.

So, multiply both sides by 2 and get $2\log\frac{1}{2} > \log\frac{1}{2}$ then using the principles of logarithm $\log\left(\frac{1}{2}\right)^2 > \log\frac{1}{2}$ so $\left(\frac{1}{2}\right)^2 > \frac{1}{2} \Rightarrow \frac{1}{4} > \frac{1}{2}$ obtained (Мадера, 2003).

Example 6. Demonstrate that $\sqrt{2(p+q)} = \sqrt{p} + \sqrt{q}$.

Let us design a triangle ABC where $BF = p$, $CF = q$, $BD = x$ and $DC = y$ are provided. $AF = h$ is the ABC triangle's height, and let $ED = h'$ be another height that divides ABC triangle's area into two equal parts. So we can express the relationship between triangular regions ABCand BED as

$$\frac{xh'}{2} = \frac{1}{2}.\frac{(p+q)h}{2} \qquad (1)$$



On the other hand, we get $\frac{h'}{h}=\frac{x}{p}$ from the similarity of the triangles BED and BAF if we solve for h' it will be $h'=\frac{hx}{p}$, and we substitute this in (1) equation to obtain $\frac{xhx}{2p}=\frac{(p+q)h}{4} \Rightarrow \frac{2x^2h}{p}=(p+q)h$ and conclude with similar approach

$$x=\sqrt{\frac{p(p+q)}{2}} \qquad (2)$$

In a similar manner, $y=\sqrt{\frac{q(p+q)}{2}}$ can be written. Knowing that $x+y=p+q$, so $\sqrt{p+q}=\sqrt{\frac{p}{2}}+\sqrt{\frac{q}{2}}$ after performing the relevant algebraic procedures, we get $\sqrt{2(p+q)}=\sqrt{p}+\sqrt{q}$ (Kondratieva, 2009).

Example 7. Prove that $\frac{a+b}{2}=a+b \quad \forall a,b \in \mathbb{R}$.

Using the binomial expansion, $(a+b)^n=a^n+na^{n-1}b+\frac{n(n-1)}{1.2}a^{n-2}b^2+...+\frac{n(n-1)}{1.2}a^2b^{n-2}+nab^{n-1}+b^n$ satisfies $\forall n \in \mathbb{N}$, when substituting for $n=2m$ we get

$$(a+b)^{2m}=a^{2m}+2ma^{2m-1}b+\frac{2m(2m-1)}{1.2}a^{2m-2}b^2+...+\frac{2m(2m-1)}{1.2}a^2b^{2m-2}+2mab^{2m-1}+b^{2m}$$

Consider the resultant expansion for $m=\frac{1}{2}$ where $a^0=1$ and $b^0=1$, and obtain $(a+b)^1=a+b+0+...+0+a+b$ so $a+b=2a+2b$ then $\frac{a+b}{2}=a+b$ (Мадера, 2003).

Example 8. Show that $2=1$.

Let's contemplate the depiction of (x^2), $x^2=x+x+x+...+x$ (x copies) for any $x \neq 0$.

Taking the derivative of both sides of the equation yields,

$$2x=1+1+1+...+1$$
$$2x=1.x$$

Dividing both sides by x, when x is not equal to zero, produces $2=1$ (Klymchuk & Staples, 2013).

Example 9. Show that $0=1$.

First, let us compute the indefinite integral $\int \frac{1}{x}dx$ using the integration formula by parts $\int udv = uv - \int vdu$ s.t. $u=\frac{1}{x}$ and $dv=dx$. This results in

$$\int \frac{1}{x}dx = \left(\frac{1}{x}\right)x - \int x\left(-\frac{1}{x^2}\right)dx = 1 + \int \frac{1}{x}dx$$

That is, $\int \frac{1}{x}dx = 1 + \int \frac{1}{x}dx$ we conclude by eliminating the identical term from both sides $0=1$ yileds (Klymchuk & Staples, 2013).

This paper can effectively operate in this context as follows. Teachers are also encouraged and advised to make their lessons lively and interesting.

a) A resource for high school and university instructors
b) A mathematics learning tool for high school and college students
c) A professional development resource for mathematics educators

The misleading strategies or erroneous thinking stages that result in sophisms are directly tied to mathematics ideas in this paper. The examples provided are designed to reinforce a correct grasp of ideas that are frequently misunderstood. When discussing traditional sophisms, we give sources wherever possible. We expect that interested academics will use the concepts presented in this study to launch future mathematical inquiries. Because it is also a valuable source of reference, this book, like the original author's earlier work, “Counterexamples in Calculus” [4], attempts to engage students and instructors to examine paradoxes and sophisms that emerge in calculus.

5. RESULTS & DISCUSSION

In mathematics, active learning involves students participating in hands-on and collaborative activities that encourage them to investigate mathematical concepts and solve problems while also preparing them to make meaningful decisions. This strategy may be useful in assisting pupils in overcoming mathematics anxiety and increasing their confidence and mathematical ability (Lugosi & Uribe, 2022).

Emotion is defined as a psychological process of expressing one's opinions about things and events in one's surroundings to other people and oneself as a cognitive and active human being, according to psychological research (Ellis et al., 2014). Emotions and feelings open new channels for accurate and extensive observation, as well as the utilization of memory capacity and the optimization of thinking and imagination, to fully comprehend the student's psychological processes, attributes, and moods. Experiment results reveal that pleasant sensations occur when students' mental activity corresponds to their age and is focused on solving specific challenges. Fear, discontent, melancholy, and other unpleasant feelings are triggered by the inability to obtain accomplishment. Emotions can increase, sustain, or decrease learning motivation and related volitional processes.

"It has been established that during the school years, the hard work and studying, and good end results of the positive activating facilitated by emotion (inner pleasure resulting from the successful solving of a task) intensify the internal motivation for achievement and stimulate learning with understanding" (Yankulova, 2012).

As a result, mathematics educators have an obligation to work toward creating an educational environment favorable to positive experiences. The teacher should be upbeat and endeavor to manage the learning process so that students can deal with the assignments since success breeds a desire and incentive to study, and learning outcomes typically improve. When the teacher's insistence and demand are followed by kindness, empathy, and a readiness to assist, the results will be obvious. Consequently, we may infer that creating a learning environment with a good emotional backdrop provides a potential to obtain higher educational outcomes in mathematics instruction (Hmelo-Silver, 2004).

It is possible to establish situations in and out of the classroom that stimulate positive sentiments, hence increasing pupils' cognitive activity. These emotions in the classroom stimulate students' thinking, connect with their cognitive interests, and help them develop themselves (Freeman et al., 2014).

Our experience demonstrates that an explanation of a mistake typically teaches the student more than a problem-free job. As a result, while teaching mathematics, it is necessary to consider using some sophisms. Students get interested in the way a task with an intentional error is completed and how it is addressed, and they become involved in the problem discussion.

The use of sophisms in mathematics instruction frequently inspires students to study and apply more mathematical material. This type of activity always adds an emotional component to the work of pupils. Students watch many scenarios, attentively analyze each stage of the task, recall concept definitions and attributes, and, as a result, recollect where the error happened (Karakasheva, 2017).

We should encourage students and teachers to examine mathematical sophisms for the following reasons.

a) To foster a deeper knowledge of concepts
b) To decrease or remove common misconceptions
c) To extend mathematical reasoning beyond procedural techniques
d) To improve essential critical thinking skills such as analysis, justification, verification, and validation
e) To increase the repertory of important mathematical ideas
f) More dynamic and inventive learning experiences for pupils
g) To encourage further investigation of mathematical issues

Mathematical sophism has played an important part in the evolution of mathematical science. This explains the first interest in the study, systematization, and instructional application of demonstrably false evidence.

6. CONCLUSION

Whether we like it or not, sophism has drawn many students and others, and it appears it will continue to do so. Since 1996, I've seen how these concerns have sparked interest in many nations, cultures, and student groups. Furthermore, several students asked, “Was everything our teachers told us a lie?” Were we taught wrong information?"

Mathematical sophism has played an important part in the evolution of mathematical science. This explains the first interest in the study, systematization, and instructional application of demonstrably false evidence. A sophism survey should not be regarded as a waste of time. There is a wide range of materials available for generating and sustaining a learning environment dominated by happy emotions. For teachers, we tried to advise various sophism assignments in the essay to assist in shaping the ideal learning environment. The intelligent and balanced use of such groups of activities not only contributes to drawing and holding student attention but also fosters learner activation, which adds to the continual learning of new mathematical information. Consequently, it is determined that mathematics is both an abstract and stereotypical topic that is mistreated and an active subject that constantly inspires individuals to make innovations.

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