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GEOMETRİK ANLAYIŞI GELİŞTİRME: OKUL MÜFREDATINDAKİ UZAKLIK KAVRAMLARINA GALİLEO GEOMETRİSİ YAKLAŞIMI

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Özet

Bu makale, Galileo geometrisi bağlamında mesafeyle ilgili kavramları öğretmeye odaklanan bir müfredatı sunmaktadır. Birincil amaç, öğrencilerin matematiksel anlayışını zenginleştirmek, bilişsel ilgiyi teşvik etmek ve temel yeterliliklerin gelişimini desteklemektir. Müfredatın merkezinde, mesafe kavramlarının iki parçalı doğasını vurgulayan Galileo geometrisi yer almaktadır. Müfredatta, "adım" ve "arşın" gibi alışılmadık birimler, tarihi ve çağdaş uzunluk ölçümlerini sergilemek ve öğrencileri farklı bakış açılarını takdir etmeye teşvik etmek için kullanılmaktadır. Müfredat, Taşkent'ten Semerkant'a olan mesafeyi ölçmek gibi gerçek dünya örnekleri kullanarak mesafe kavramlarını tanıtmaktadır. Şehirlerarası yolculuk planlaması gibi pratik uygulamalar, bu birimlerin günlük senaryolardaki faydasını vurgulamaktadır. Çağdaş bilgi teknolojisi ve kodlama yöntemleri, mesafe kavramlarının çağdaş bağlamlardaki önemini göstermek için entegre edilmiştir. Müfredat, ileri düzey konulara girerek Minkowski geometrisini incelemekte ve Öklid geometrisiyle karşılaştırmalar yapmaktadır. Pratik uygulama bölümleri, öğrencilerin anlayışını değerlendirmek için sorular içermektedir ve makalede sunulan tablo, müfredattaki bilgilerinin izlenmesini ve iyileştirilecek alanların belirlenmesini kolaylaştırmaktadır. Önerilen bağımsız bir görev, öğrencileri mesafeyle ilgili kavramlar hakkındaki bilgilerini pekiştirmeye teşvik etmektedir. Müfredat, iş birlikli öğrenmeyi ve eleştirel düşünmeyi vurgulayan bir 'Boomerang' yöntemi ile sona ermektedir. Önerilen ek materyaller, tartışma grupları ve atanan görevlerin öğrencilerin Galileo geometrisi ve uygulamalarına ilişkin kavrayışını güçlendireceği umulmaktadır. Gelecek çalışmalarda, bu müfredatın yalnızca Galileo geometrisinin temel yönlerini ele almakla kalmadığı için eleştirel düşünmeyi, iş birlikçi öğrenmeyi ve geometrinin bütünsel bir anlayışını da teşvik edip etmediğinin araştırılması önerilmektedir.

Anahtar Kelimeler: Eleştirel düşünme, gerçek dünya uygulamaları, Galileo geometrisi, müfredat tasarımı, uzaklık kavramları.

ENHANCING GEOMETRIC UNDERSTANDING: A GALILEAN GEOMETRY APPROACH TO DISTANCE CONCEPTS IN SCHOOL CURRICULUM

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Abstract

This article presents a curriculum focused on teaching distance concepts in the context of Galilean geometry. The primary goal is to enhance students' mathematical understanding, stimulate cognitive interest, and support the development of core competencies. At the core of the curriculum is Galilean geometry, which emphasizes the two-part nature of distance concepts. Unconventional units such as "step" and "cubit" are used throughout the curriculum to showcase historical and contemporary length measurements and encourage students to appreciate different perspectives. The curriculum introduces distance concepts using real-world examples, such as measuring the distance from Tashkent to Samarkand. Practical applications, such as planning intercity trips, highlight the utility of these units in everyday scenarios. Contemporary information technology and coding methods are integrated to demonstrate the importance of distance concepts in contemporary contexts. The curriculum delves into advanced topics, examining Minkowski geometry and making comparisons with Euclidean geometry. Practical application sections include questions to assess students' understanding, and the table presented in the article facilitates monitoring of their knowledge across the curriculum and identifying areas for improvement. A proposed independent task encourages students to consolidate their knowledge of distance-related concepts. The curriculum concludes with a 'Boomerang' method that emphasizes collaborative learning and critical thinking. It is hoped that the proposed supplementary materials, discussion groups and assigned tasks will strengthen students' understanding of Galilean geometry and its applications. Future studies are recommended to investigate whether this curriculum not only addresses the fundamental aspects of Galilean geometry but also encourages critical thinking, collaborative learning and a holistic understanding of geometry.

Keywords: Critical thinking, curriculum design, distance concepts, Galilean geometry, real-world applications.

1. INTRODUCTION

In contemporary education, the pursuit of effective teaching methodologies that foster profound conceptual understanding and engagement among students is an ongoing endeavor. Geometry, as a fundamental branch of mathematics, often poses challenges for learners due to its abstract nature. This paper explores the potential of collaborative learning, specifically employing the "Boomerang" method, as a pedagogical tool to enhance the mastery of Galilean Geometry concepts among secondary school students.

The geometry curriculum not only equips students with essential mathematical skills but also cultivates critical thinking and problem-solving abilities. However, traditional instructional

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approaches may fall short in addressing the diverse learning needs and preferences of students. Collaborative learning, characterized by active student interaction and shared responsibility for learning outcomes, emerges as a promising alternative to conventional teaching methods.

“The Boomerang” method, a structured collaborative learning approach, is designed to promote active participation, mutual support, and knowledge transfer among students. This study aims to investigate the impact of this method on students' conceptual grasp of Galilean Geometry principles and compare its effectiveness against traditional teaching methods. The findings contribute valuable insights to the broader discourse on innovative pedagogical practices and their role in shaping a more dynamic and engaging learning environment.

2. REVIEWED LITERATURE

Educational literature has witnessed a growing interest in collaborative learning as a means to enhance student outcomes across various disciplines. Theoretical frameworks such as social constructivism highlight the importance of social interaction and shared experiences in the learning process (Vygotsky, 1978). Collaborative learning environments are believed to foster active engagement, deeper understanding, and the development of interpersonal skills (Johnson & Johnson, 1989).

In the specific context of geometry curriculum, studies have explored the efficacy of collaborative learning in improving student performance and conceptual mastery. Artiqbayev (2004) emphasized the significance of integrating modern teaching methods to address the evolving needs of students in geometry curriculum. “The Bumerang” method, as an innovative approach, aligns with this objective by creating a structured yet dynamic platform for collaborative exploration of geometric concepts.

Research by Khachaturyan (2005) delves into Galilean Geometry, highlighting its unique principles and applications. However, the literature gap lies in the exploration of pedagogical strategies tailored to optimize the understanding of these intricate concepts. This study addresses this gap by focusing on the “Boomerang” method and its potential to enhance Galilean Geometry comprehension.

We also want to mention the studies (Kurudirek & Akca, 2015; Kurudirek, 2022; Kurudirek, 2023) that have greatly assisted us in this matter. Especially, we have endeavored to progress in the light of studies filled with teaching methods for non-Euclidean geometries to middle and high school students and information about these interesting geometries.

In the realm of secondary education, Haydarov et al. (2019) emphasize the need for engaging and effective teaching practices in geometry classrooms. Collaborative learning approaches have shown promise in addressing this need by promoting active participation and a supportive learning community.

In conclusion, this paper contributes to this body of literature by presenting empirical evidence on the impact of “the Boomerang” method in a secondary school geometry setting.

2.1. Enhancing Geometry Understanding

Extracurricular activities play a pivotal role in enriching students' mathematical experiences beyond the confines of traditional classroom settings. These activities serve various educational objectives, including broadening students' perspectives, fostering a deeper appreciation for mathematics, nurturing cognitive interest, and honing practical application skills. This paragraph explores how distance-related concepts can be effectively integrated into extracurricular activities to achieve these educational goals.

In the realm of mathematics, understanding the significance of distance is foundational. An engaging approach involves posing real-world questions such as "How far is it from Tashkent to Samarkand?" Drawing inspiration from the article "Ancient Subject in a Modern Perspective" (Artiqbayev, 2004), this activity prompts students to explore the concept of distance in a straight line. By replacing the cities with others, this exercise emphasizes that distance transcends mere length, introducing students to historical units like "step," "pace," "span," "foot," and "ell." This historical context not only enriches their knowledge but also instills a sense of curiosity about mathematical concepts.

Galilean geometry provides a fertile ground for optional activities aimed at enhancing students' geometric perceptions and scientific engagement. One noteworthy aspect of Galilean geometry is the dual nature of the distance concept. Practical examples, such as travel planning between cities, vividly illustrate the utility of these distances in real-life scenarios. Explaining how a traveler moves from one city to another highlights the intricacies of Galilean geometry, showcasing its relevance beyond theoretical constructs. Introducing alternative city pairs further reinforces the understanding that distance encompasses more than just length, fostering a non-traditional perspective.

Incorporating historical units of measurement into the discussion, such as "step," "pace," "span," "foot," and "ell," not only connects students with the roots of measurement but also adds a layer of historical intrigue to the subject. This multifaceted approach serves to deepen students' interest in Galilean geometry and presents mathematical concepts within a broader historical and practical context.

Once students grasp the fundamentals of distance in Galilean geometry through practical examples, introducing the distance formula in Minkowski geometry becomes a compelling next step. This advanced concept not only captivates students' interest but also aligns with modern geometric theories, notably Einstein's theory of relativity. Encouraging independent study of the distance formula empowers students to enhance their analytical skills and delve into the fascinating intersection of geometry and relativity (Atanasyan, 2001).

These recommendations for incorporating distance-related concepts into extracurricular activities not only provide a practical guide for optional sessions but also hold potential applicability in general curriculum school mathematics classes. Emphasizing the diversity of the distance concept, its deviation from conventional Euclidean geometry, and shedding light on the historical origins of geometric concepts can stimulate students' curiosity, encourage further exploration, and foster critical thinking skills. Participation in such facultative activities not only augments mathematical knowledge but also nurtures students' logical thinking and critical reasoning abilities.

Overall, the literature review underscores the relevance of collaborative learning in geometry curriculum and positions the “Boomerang” method as a novel intervention worthy of empirical investigation. This research aims to build upon existing knowledge by providing empirical insights into the effectiveness of this method in enhancing the mastery of Galilean Geometry concepts among secondary school students.

3. METHODOLOGY

3.1. Research Design

The study adopts an experimental approach, employing a quasi-experimental design to assess the effectiveness of the collaborative learning activity based on the “Boomerang” method. This design allows for a comparison between the experimental group, participating in the collaborative activity, and a control group undergoing traditional teaching methods (Ary et al., 2010; Khoirunnisa & Cahyani, 2023).

3.2. Participants

48 students of the specialized school in the Paxtakor district, supervised by the educational administration of the Jizzakh region, from primary to secondary curriculum.

The study involves 48 students of the specialized school in the Paxtakor district, supervised by the educational administration of the Jizzakh region, from primary to secondary curriclum. The participants are divided into two groups: the experimental group and the control group. Group allocation is done randomly to ensure unbiased representation.

3.3. Activities, Materials and Instruments

For students participating in facultative activities, the teacher provides information about distance-related concepts in Galilean geometry:

The distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ is calculated by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (1)$$

If $d = \varphi(A, B)$ is defined, $\varphi(A, B)$ is called the distance function. Since $\varphi(A, B) = \varphi(B, A)$, the distance function is symmetric (Kurudirek & Akca, 2015; Kurudirek, 2023). Now, let's assume that we have a Cartesian coordinate system $O'x'y'$ in β plane under certain conditions. If we consider the planes α and β as one plane, the following reflection can be established between the Cartesian coordinate system in them, that is, between the coordinates of points corresponding to each other:

$$f: \begin{cases} x' = x\cos\alpha - y\sin\alpha + a \\ y' = x\sin\alpha + y\cos\alpha + b \end{cases} \qquad (2)$$

If points A and B are reflected to points A' and B'in f reflection, it can be proved that the distance (1) between them is $\varphi(A, B) = \varphi(A', B')$. From this, the length of the cross-section determined by equation (1), that is, the distance (2), is preserved in the reflection. So there is an isometry between the planes. School geometry is a science that studies the properties of plane shapes (2) preserved in reflection. Plane geometry (planimetry), which is usually studied at school, is called Euclidean geometry.

For students to get acquainted with Galilean geometry, school mathematics classes should have information about the plane coordinate system. So, we assume that the Oxy Cartesian coordinate system is installed on the plane, and each point on the plane has its own pair of coordinates. We determine the distance between two points on the plane in the following way.

We are given α plane and the Oxy coordinate system is set on it. In this case, any point of the plane will have its own pair of (x, y) coordinates.

The length d of the line segment with its ends at points $A(x_1, y_1)$ va $B(x_2, y_2)$ is given by,

$$d = |x_2 - x_1|$$

If $d = 0$, namely $x_1 = x_2$ then the distance is given by,

$$d = |y_2 - y_1|$$

If we write this correspondence more precisely,

$$\varphi(A, B) = d = \begin{cases} |x_2 - x_1|, & if \ \ x_1 \neq x_2 \\ |y_2 - y_1|, & if \ \ x_1 = x_2 \end{cases} \tag{3}$$

This input quantity can be the distance between two points, since $\varphi(A, B) = \varphi(B, A)$. Now we need to show that there exists a reflection that preserves this distance.

Hereby,

$$\begin{cases} x' = x + a \\ y' = h \cdot x + y + b \end{cases} \tag{4}$$

distance is maintained in reflection (3). Indeed, if in reflection (4) points A and B shift to points A' and B' respectively then,

$$\varphi(A', B') = |x_2' - x_1'| = |x_2 + a - (x_1 + a)| = |x_2 - x_1| = \varphi(A, B)$$

If $x_1 = x_2$ then,

$$\varphi(A', B') = |y_2' - y_1'| = |hx_2 + y_2 + b - (hx_1 + y_1 + b)| =$$

$$= |h(x_2 - x_1) + y_2 - y_1| = |y_2 - y_1| = \varphi(A, B)$$

In both cases, the distance between the corresponding points does not change. So, (4) is an action to replace. As long as the distance entered in the plane (3) is preserved in the reflection (4). So this is geometry. This resulting geometry is called Galilean geometry.

In order to strengthen the concepts related to the learned distance, to determine to what extent the learned knowledge has been mastered, at the end of the optional training, a handout will be distributed to the students to fill in the table below with the help of insert technology.

To reinforce the concepts related to distance, as well as to assess the level of knowledge acquired, additional materials are provided at the end of the facultative activity using the insert technology (Kurudirek, 2022). A table is filled out by students to evaluate their understanding of distance-related concepts. The table includes questions such as:

Concepts related to distance in Galilean geometry	V	+	-	?
1. Do you know about the distance in Galilean geometry?				
2. How many parts is the distance in Galilean geometry composed of and why?				
3. Do you have information about the concept of isometry?				
4. Do you know the modern definition of the geometry discipline?				
5. Can you provide a real-life example related to distance in Galilean geometry?				
6. Can you find the distance between points A(1, 0) and B(5, 8)?				
7. Can you find the distance between points A(-5, 5) and B(-5, 10)?				
8. Do you understand the representation of motion in Galilean geometry?				

The symbols in the table indicate the following meanings:

(V) - I know.
(+) - New information for me.
(-) - I know but deny it.
(?) - Uncertain, ambiguous information.

The table above is presented to each student, and each student completes this table. The teacher collects and analyzes these tables. As a result of the analysis, information about the extent to which students have mastered these concepts is obtained. If the “I know” indicator in the results table is lower than other indicators, in the next activity, it is definitely required to repeat and consolidate the acquired knowledge.

To reinforce and consolidate the acquired knowledge, an activity based on the “Boomerang” method can be organized as follows:

In the main part of the activity: Each group is given additional material (advisably, when forming groups with 24 students, divide them into 4 groups. In each group, each student must be able to answer at least one question related to the “Insert” technique). They prepare in 10 minutes for the questions in their supplementary material. In this case, each student who answered “I know” to at least one question in the group should teach the concept to other members of the group.

Then, from the 1-numbered students of each group, another separate group is formed. Similarly, groups are formed from 2-numbered students, 3-numbered students, and so on, until groups are formed from 6-numbered students. Thus, a total of 6 new groups are formed.

Now, each student in the newly formed 6 groups discusses the above questions together. They are given an additional 10 minutes for this.

After the specified time, each student from each group other than the 1-numbered students writes down and protects the tasks on the board according to the teacher's recommendation. Other students may ask questions.

- Supplementary Material: Tailored material related to Galilean Geometry is provided to each group, consisting of challenging questions aligned with the curriculum.
- Questionnaires: Each student completes a pre-activity questionnaire to gauge their initial understanding.
- Scoring Criteria: A detailed rubric is established to evaluate individual and group performances during the collaborative learning activity.
- Reference Materials: Existing textbooks and academic resources on the subject matter are referred to during the activity and as supplementary resources.

3.4. Procedure

The procedures followed were as the following step by step:

- Pre-Activity Assessment: Prior to the collaborative learning activity, a pre-activity questionnaire is administered to both groups to assess their baseline understanding of Galilean Geometry concepts.
- Formation of Groups: In the experimental group, students are divided into smaller groups of four based on the “Insert” technique. Each student in a group must have answered “I know” to at least one question in the supplementary material.
- Collaborative Learning Activity: The “Boomerang” method is implemented, with each group working collectively to prepare and discuss the supplementary material. This involves peer teaching, knowledge reinforcement, and concept clarification within each group.
- Post-Activity Assessment: After the collaborative learning activity, a post-activity questionnaire is administered to both groups to evaluate the extent of concept mastery.
- Board Presentation: Each student, excluding the initial responders, presents and defends the solutions on the board. Interaction is encouraged, allowing group members to ask questions.
- Scoring and Evaluation: Performance is scored based on predetermined criteria, considering individual and collaborative achievements. The rubric includes categories such as completeness, correctness, and independence of responses.

As an independent work for students, the following case assignment can be given as homework to check the acquired knowledge about concepts related to distance in Galilean geometry:

Case 1:

Give us a uniqueness α, and let the Oxycoordinate-system be installed on it. In this case, any point of uniqueness will have its own pair (x, y) coordinates."

The length d of the line segment with its ends at points $A(x_1, y_1)$ va $B(x_2, y_2)$ is given by,

$$d = |x_2 - x_1|$$

If $d = 0$, namely $x_1 = x_2$ then the distance is given by,

$$d = |y_2 - y_1|$$

If we write this correspondence more precisely,

$$\varphi(A, B) = d = \begin{cases} |x_2 - x_1|, & if \;\; x_1 \neq x_2 \\ |y_2 - y_1|, & if \;\; x_1 = x_2 \end{cases} \qquad (3)$$

This input quantity can be the distance between two points, since $\varphi(A, B) = \varphi(B, A)$.

Case 1 assignment. Indicate the general and specific aspects of distance in Galilean geometry compared to Euclidean geometry, considering the concept of distance in Galilean geometry.

Case 2:

Definition. Let's consider a set consisting of either finite or infinite elements, denoted as X. Let's refer to its elements as points. Suppose a function $\varphi(x, y) = k$ is given that assigns a real number to each pair of elements $x, y \in X$. If the function $\varphi(x, y)$ satisfies the condition $(x, y) = \varphi(y, x)$, we designate the real number k as the "distance" between points $x, and\ y$.

Now, let's assume we have another set Y. There exists a mapping f that associates elements of X and Y in such a way that the distances between corresponding points are preserved, meaning,

$\varphi(x, y) = \varphi(f(x), f(y))$.

Sets X and Y, equipped with such a matching, are referred to as isometric sets. We denote all sets isometric to X as $Y = IsomX$. The contemporary description of geometry introduced by Thurston is as follows:

Definition: A field of study that investigates properties related to sets $(X,\ IsomX)$ is called geometry.

Assignment: Demonstrate how the geometry being taught in school aligns with the definition mentioned above.

Table 1. Rubric

Criteria	Score
if all the tasks in the questionnaire are complete and correctly solved, and full answers are given to the questions asked	5 points
if the tasks are completed correctly, partial shortcomings are present in the explanation, and full answers are given to the questions asked	4 points
if the tasks are completed with the help of others, difficulties in explaining are observed, and the answers to the questions asked are incomplete	3 points
if the tasks are completed with the help of others, difficulties in explaining are observed, and the answers to the questions asked are inadequate	2 points
if the tasks are completed with the help of others, assistance from group members is noted, and another group member helps to answer the questions	1 point

Additionally, an additional point is awarded to the student in the group who helped with the board tasks to encourage involvement.

- Feedback and Homework Assignment: Feedback is provided, and participants are given homework assignments to reinforce learning.

3.5. Data Analysis

Quantitative data collected from pre- and post-activity questionnaires are analyzed using statistical methods to compare the improvement in understanding within the experimental and control groups. Qualitative data from board presentations and interactions are analyzed to identify collaborative learning dynamics, understanding transfer, and areas of improvement.

3.6. Ethical Considerations

Informed consent is obtained from participants. The anonymity and confidentiality of participants are ensured. The study complies with ethical standards and guidelines of the Jizzakh region's Department of Primary and Secondary Education for January 8, 2024, with serial number 01-16.

The methodology combines quantitative and qualitative approaches to comprehensively assess the impact of the collaborative learning activity on enhancing concept mastery in Galilean Geometry. The chosen design and instruments aim to provide valuable insights into the effectiveness of the “Boomerang” method as an instructional strategy.

4. FINDINGS

In the boomerang method, a higher degree of development in critical thinking, logic-based reasoning, and presentation of evidence skills was identified among students in the experimental group who were taught knowledge related to "Galilean geometry." During the activities in the experimental group, the teacher's active involvement in demonstrating acquired knowledge through oral and written explanations, as well as developing skills in expression and understanding during exercises, contributed to the students' increased ability to critically analyze.

The teaching method, in conjunction with the curriculum, allowed for the implementation of the following series of curriculum tasks with a character of upbringing:

- Teamwork skills;
- Adaptability to interpersonal communication;
- Positivity;
- Cooperation;
- Respect for others' opinions;
- Activeness;
- Creative orientation in one's work;
- Self-evaluation.

This approach provided an opportunity for students to not only acquire subject-specific knowledge but also to develop important personal and interpersonal skills.

4.1 Experimental Group Outcomes

The experimental group, immersed in the collaborative learning experience facilitated by the "Boomerang" method, demonstrated a remarkable enhancement in overall performance compared to their counterparts in the control group. Pre- and post-activity assessments unveiled a substantial increase in the depth of comprehension regarding Galilean Geometry concepts.

Qualitative analysis of the collaborative learning sessions revealed intricate dynamics within the experimental group. Peer teaching, active discussions, and collective problem-solving were recurrent themes, underscoring a robust engagement with the subject matter. The collaborative environment not only improved conceptual understanding but also fostered a sense of shared responsibility for learning outcomes.

4.2 Control Group Comparisons

In contrast, the control group, exposed to conventional instructional methods, exhibited a comparatively modest improvement. Traditional teaching approaches seemed less effective in stimulating the same level of active participation, dynamic discussions, and mutual support observed in the collaborative learning setting.

Members of the control group demonstrated a more restricted depth of understanding concerning Galilean Geometry principles. The absence of collaborative activities appeared to impact their ability to grasp intricate concepts comprehensively. This finding highlights the potential limitations of traditional teaching methods in fostering a deep conceptual understanding.

4.3. Participants' Experiences

Theme 1. Positive Learning Experience: Qualitative feedback from participants in the experimental group echoed a positive learning experience. Collaborative learning, as facilitated by the "Boomerang" method, was perceived as enriching and engaging. Active interaction with peers not only enhanced their understanding of the subject but also contributed to a more enjoyable and interactive learning environment.

Theme 2. Increased Confidence: An interesting outcome was the reported increase in students' confidence levels. Collaborative learning provides a supportive platform for students to express their ideas and articulate complex concepts. The shared responsibility for group outcomes appeared to contribute to a boost in self-assurance.

4.4. Comparative Analysis

The findings underscore the effectiveness of active participation and collaborative problem-solving compared to more passive learning approaches. Collaborative learning methodologies, particularly the "Boomerang" method, actively involve students in the learning process, fostering a deeper conceptual grasp and higher-order thinking skills.

An intriguing observation was the higher degree of knowledge transferability exhibited by participants in the experimental group. The collaborative approach not only enhanced their understanding of isolated concepts but also facilitated the application of knowledge to novel problem-solving scenarios. This suggests that collaborative learning may contribute to a more holistic and versatile comprehension of the subject matter.

5. DISCUSSION

The study sought to investigate the efficacy of a collaborative learning activity, grounded in the "Boomerang" method, in enhancing students' understanding of Galilean Geometry concepts. Results from both quantitative and qualitative analyses provide valuable insights into the impact of this instructional strategy.

Quantitative data revealed a statistically significant improvement in the experimental group's performance compared to the control group, confirming earlier research (Fahri et al., 2016; Khoirunnisa & Cahyani, 2023). Pre- and post-activity questionnaires demonstrated a notable increase in the mastery of Galilean Geometry concepts among students engaged in collaborative learning.

In line with the literature, the qualitative analysis of board presentations and group interactions shed light on the collaborative learning dynamics (Head, 2003; Meirink, 2009). Peer teaching, active discussions, and collective problem-solving were evident in the experimental group, fostering a deeper understanding of the material.

5.1. Implications

The findings hold significant implications for pedagogy, emphasizing the potential benefits of incorporating collaborative learning activities into curriculum practices. Specifically, the "Boomerang" method emerges as a promising tool for enhancing student engagement, conceptual understanding, and knowledge retention in the field of geometry curriculum. Educators are encouraged to explore and integrate collaborative learning strategies to optimize student learning outcomes in similar academic settings.

The study underscores the pedagogical benefits of collaborative learning activities, particularly those structured around the "Boomerang" method. Encouraging students to actively engage with course material through peer collaboration enhances critical thinking, communication skills, and conceptual understanding.

Integrating collaborative learning activities into the curriculum can be an effective strategy for educators. The findings suggest that such activities contribute to a more interactive and engaging learning environment, potentially improving overall academic performance.

The collaborative approach fosters a sense of shared responsibility and engagement among students. Beyond the specific subject matter, the experience may positively impact students' attitudes toward learning and their peers.

5.2. Limitations and Further Research

As limitations, external factors such as individual differences in prior knowledge and learning styles may impact results. The study's generalizability may be constrained to the specific context and subject matter.

The study paves the way for future investigations into the long-term impact of collaborative learning on knowledge retention and its applicability across various geometrical concepts. Additionally, exploring the scalability of the "Boomerang" method in different curriculum contexts and subjects is warranted. Further research should delve into refining collaborative learning strategies to suit diverse learning environments and student profiles.

Exploring how different learning styles interact with collaborative learning methods could provide nuanced insights. Understanding how individuals with diverse learning preferences engage in group activities may guide instructional strategies tailored to specific cohorts.

Investigating the long-term effects of collaborative learning on knowledge retention and application is essential. Follow-up studies could assess whether the observed improvements in understanding persist over time and influence subsequent academic performance.

Examining the applicability of collaborative learning models in various academic disciplines may reveal subject-specific nuances. Adapting collaborative strategies to align with the unique characteristics of different subjects could optimize their effectiveness.

6. CONCLUSION

In conclusion, this study contributes to the growing body of research supporting the integration of collaborative learning activities, specifically those grounded in the "Boomerang" method, as effective tools for enhancing concept mastery. The implications for pedagogy and suggestions for further research provide a foundation for educators and researchers to continue exploring innovative approaches to student-centered learning.

REFERENCES

Atanasyan, L. S. (2001). *Geometry of Lobachevsky*. Enlightenment.

Artiqbayev, A. (2004). Modern view on ancient knowledge. *Journal of Physics, Mathematics, and Informatics*, *3*(1), 3-17.

Ary, D., Jacobs, L. C., Sorensen, C., & Razavieh, A. (2010). *Introduction to research in education*. Wadsworth/Thomson Learning Group.

Fahri, S., Abdullah, K., & Huseyin, A. (2016). Determination of efficiency of educational work of class teachers in comprehensive school. *European Research*, *8*(19), 92-94.

Haydarov, B., Sariqov, E., & Qo'chqorov, A. (2019). *Geometry – 9th grade: Textbook for students of general secondary schools of the 9th grade*. Tashkent, Uzbekistan: O'qituvchi.

Head, G. (2003). Effective collaboration: Deep collaboration as an essential element of the learning process. *The Journal of Educational Enquiry*, *4*(2).

Khoirunnisa, A. S., & Cahyani, I. (2023). Promoting speaking skills through engage, study, activate (ESA) learning model for BIPA learners. *International Seminar on Language, Education, and Culture (ISoLEC)*, *7*(1), 381-388.

Khachatryan, A. V. (2005). Galilean geometry. *Library of Mathematical Enlightenment*, *4*(1), 1-18.

Kurudirek, A. (2022). Methods of using non-Euclidean geometry concepts in the educational process. *Bulletin of the Institute of Mathematics*, *5*(1), 1-5.

Kurudirek, A. (2023). Illuminating high school students with some interesting non-Euclidean geometries in the plane. *International Journal of Statistics and Applied Mathematics*, *8*(4), 63-68. https://doi.org/10.22271/maths.2023.v8.i4a.1100

Kurudirek, A., & Akca, H. (2015). Explanation of distance, kinematic and isometry to high school students from different perspective. *International Research in Education*, *3*(1), 103-110. https://doi.org/10.5296/ire.v3i1.7036

Meirink, J. A., Meijer, P. C., Verloop, N., & Bergen, T. C. (2009). Understanding teacher learning in secondary education: The relations of teacher activities to changed beliefs about teaching and learning. *Teaching and Teacher Education*, *25*(1), 89-100.